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An Analytical Reassessment of Olbers' Paradox: FLRW Cosmology and a Conditional Propagation-Redshift Toy Model

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

Olbers' paradox is a useful convergence test for cosmological radiation fields because an infinite, homogeneous, eternal and static population of identical luminous sources produces a divergent bolometric flux in Euclidean space. This analytical study compares that classical result with the standard Friedmann-Lemaitre-Robertson-Walker (FLRW) background-intensity formulation and a deliberately restricted propagation-redshift toy model. In FLRW cosmology, the observed bolometric background follows from comoving luminosity density, luminosity distance, comoving volume and the expansion history; a finite source history, source evolution, cosmological redshift and geometry jointly invalidate the assumptions of the classical static construction. For the alternative toy model, the local postulate $d \ln \lambda / ds = c^{-1} |du/dr|$ is examined for $u(r) = -\sqrt{2GM/r}$. The resulting single-domain redshift approaches a finite asymptote and therefore cannot regularise the

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classical Olbers integral for any finite positive flux-redshift exponent q. A deterministic cumulative ansatz, $1 + z = \exp(\alpha r)$, is then analysed only as a conditional mathematical construction. It yields $F = nL/(\kappa + q\alpha)$ when $\kappa + q\alpha > 0$. A new stochastic-domain extension shows that a positive mean logarithmic redshift increment alone is insufficient for ensemble-mean convergence: for independent increments $Y = \ln(1 + \delta)$, the relevant quantity is $m_q = E[e^{-qY}] = E[(1 + \delta)^{-q}]$. Without attenuation, ensemble-mean convergence requires $m_q < 1$ in both fixed-density and Poisson domain-count models. The model is further confronted analytically with luminosity-distance curvature, supernova time dilation, Tolman surface-brightness dimming, baryon acoustic oscillation distance information, CMB spectral preservation and extragalactic background-light constraints. These comparisons reinforce the conclusion that the propagation construction is a falsifiable toy model rather than an observationally established alternative to FLRW/ΛCDM.

Keywords: Olbers' paradox; FLRW cosmology; cosmological redshift; luminosity distance; extragalactic background light; photon propagation; convergence; stochastic domains.

1 Introduction

Olbers' paradox asks why the night sky is dark if luminous sources are distributed throughout an infinite, homogeneous, eternal and static universe. In the idealised Euclidean construction, the number of sources in a spherical shell grows as r^2, whereas the received flux from each source falls as r^{-2}. The two factors cancel, so equal radial intervals contribute equal amounts of flux and integration to arbitrarily large radii diverges. Harrison (1964) and Wesson et al. (1987) emphasised that the historical resolution is not captured by the slogan that expansion alone makes the sky dark. Finite source lifetimes and the finite age of luminous populations are central, while cosmological redshift and the geometry of an expanding spacetime provide additional suppression. Measurements of the extragalactic background light (EBL) likewise interpret the diffuse radiation field as the integrated outcome of source formation, evolution, propagation and cosmological geometry (Dwek and Krennrich, 2013; Hauser and Dwek, 2001).

The standard cosmological description uses an expanding FLRW spacetime whose parameters are constrained simultaneously by several independent observations, including the cosmic microwave background (CMB), Type Ia supernovae, baryon acoustic oscillations (BAO) and large-scale structure (Alam et al., 2021; Brout et al., 2022; Frieman et al., 2008; Planck Collaboration, 2020). The luminosity-distance relation is tied to the angular-diameter distance through the distance-duality relation under the usual assumptions of metric propagation and photon-number conservation (Etherington, 2007). Consequently, a non-expansion redshift prescription must do substantially more than make an Olbers-type integral converge. It must independently reproduce the observed redshift-distance relation, luminosity distance, time dilation, surface-brightness evolution, background spectra and opacity constraints.

A velocity-differential redshift mechanism was proposed by Ranzan (2014) within a non-standard, non-expanding cellular cosmology. That paper explicitly invokes a luminiferous substrate and is not part of the established FLRW/General Relativity framework. The present study does not adopt those broader physical claims. It isolates only a mathematical wavelength-evolution postulate, applies it to a specified radial velocity field, and asks what follows analytically. This distinction is essential: a mathematical convergence condition can be internally correct without establishing a viable physical mechanism.

However, a clear analytical gap remains between demonstrating a local wavelength-evolution law and establishing convergence of the integrated radiation background, particularly when flux suppression, cumulative multi-domain effects, stochasticity and independent observational discriminants are treated consistently. The present study addresses this gap by separating these issues within a common analytical framework.

The paper has six objectives. First, the classical static divergence is restated as a precise benchmark. Second, the corresponding bolometric FLRW background-intensity expression is formulated using cosmological distance and volume measures rather than a Euclidean shell with an appended redshift factor. Third, the single-domain velocity-differential postulate is solved analytically and its directional and energy-accounting implications are made explicit. Fourth, the flux mapping is written with an independent exponent q so that energy loss and arrival-rate dilation are not conflated. Fifth, the deterministic cumulative law is supplemented with a stochastic-domain treatment that distinguishes typical-path growth from ensemble-mean flux convergence. Sixth, the resulting toy model is compared with modern observational constraints through analytic discriminants. No global cosmological fit is attempted; the contribution is a mathematical reassessment and a set of explicit falsifiability conditions.

2 Scope, Assumptions and Notation

Three conceptually distinct frameworks are compared and must not be mixed.

1. **Classical Static Benchmark:** Space is Euclidean, sources have constant physical number density n and luminosity L, and source activity extends without a high-distance or finite-age cutoff.
2. **FLRW Benchmark:** Geometry, redshift, luminosity distance, comoving volume and source history are treated self-consistently in an expanding spacetime. The relevant emissivity is the comoving bolometric luminosity density $\mathcal{L}(z)$.
3. **Static Propagation Toy Model:** Geometry remains Euclidean while photon propagation is assigned a phenomenological redshift $z(r)$ and optional effective attenuation $\tau(r)$. This is not presented as a derivation from General Relativity, Maxwell theory or a microscopic interaction.

In the propagation model, q denotes the exponent with which redshift suppresses bolometric flux. A mechanism that only lowers photon energy corresponds to $q = 1$ if photon arrival intervals are unchanged. If the same mechanism also stretches arrival intervals by an additional factor $(1 + z)$, the bolometric suppression becomes $q = 2$. More general positive q values are retained mathematically, but any physical interpretation requires a specific mechanism. The parameter $\kappa \geq 0$ is an effective attenuation coefficient in $\tau(r) = \kappa r$. Because ordinary absorption can be followed by re-radiation, κ is not regarded as an independent bolometric solution of Olbers' paradox in an eternal equilibrium universe.

For the cumulative-domain construction, the ith domain is assigned a multiplicative wavelength factor $1 + \delta_i > 0$. Defining $Y_i = \ln(1 + \delta_i)$ makes the cumulative logarithmic shift additive. The domain-count function is $N(r)$. A deterministic density model uses $N(r) \simeq \eta r$, whereas a stochastic count model may take $N(r)$ to be Poisson with mean ηr. These assumptions are introduced solely to examine convergence; they are not inferred from the single-domain field.

3 Classical Static Benchmark

Consider identical sources of luminosity L with constant physical number density n in an infinite, static Euclidean space. The received bolometric flux from one source at distance r is

$$F_{\text{single}}(r) = \frac{L}{4\pi r^2}.$$

The number of sources in a shell of thickness dr is

$$dN = 4\pi n r^2\, dr.$$

The shell contribution is therefore

$$dF_{\text{shell}} = F_{\text{single}}\, dN = nL\, dr,$$

and integration to arbitrarily large radii gives

$$F_{\text{static}} = nL \int_0^{\infty} dr \to \infty.$$

The divergence follows only under the combined assumptions of infinite spatial extent, eternal source activity, static Euclidean geometry, homogeneous non-zero emissivity and no physically effective high-distance cutoff. The observed universe does not satisfy this full set of assumptions. In particular, luminous sources have a finite formation history, and the mapping among source luminosity, redshift, distance, volume and observed flux is not Euclidean in FLRW cosmology (Harrison, 1964; Wesson et al., 1987).

4 Bolometric Background Intensity in FLRW Cosmology

4.1 Expansion History and Distance Measures

For a homogeneous and isotropic FLRW spacetime, the expansion rate can be written

$$H(z) = H_0 \left[\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda\right]^{1/2}.$$

The luminosity distance D_L and transverse comoving distance D_M satisfy

$$D_L = (1+z) D_M.$$

This relation is consistent with the Etherington distance-duality relation $D_L = (1+z)^2 D_A$ and $D_M = (1+z) D_A$ under the usual metric assumptions (Etherington, 2007). The comoving volume element per redshift interval and solid angle is

$$\frac{dV_c}{dz\, d\Omega} = \frac{c D_M^2}{H(z)}.$$

These relations are not optional corrections to a Euclidean calculation. Together they define how source luminosity and source counts map to the observed radiation field in an expanding spacetime (Frieman et al., 2008; Planck Collaboration, 2020).

4.2 Comoving Emissivity Formulation

Let $\mathcal{L}(z)$ be the rest-frame comoving bolometric luminosity density, with units of luminosity per comoving volume. Let $\tau(z)$ represent an optional effective optical depth for the observational quantity being considered. For a strictly bolometric calculation in which absorbed energy is ultimately re-radiated and retained in the energy census, interpreting τ as a permanent sink would be inappropriate; for a band-limited background, however, effective attenuation can be relevant.

The luminosity in a comoving shell is $\mathcal{L}(z) dV_c$. Dividing the observed flux by the shell solid angle gives

$$dI_{\rm bol} = \frac{\mathcal{L}(z)}{4\pi D_L^2} \frac{dV_c}{d\Omega} e^{-\tau(z)} = \frac{c}{4\pi} \frac{\mathcal{L}(z) e^{-\tau(z)}}{H(z)(1+z)^2}\, dz.$$

Hence

$$I_{\rm bol} = \frac{c}{4\pi} \int_0^{z_{\rm max}} \frac{\mathcal{L}(z) e^{-\tau(z)}}{H(z)(1+z)^2}\, dz.$$

The source history resides in $\mathcal{L}(z)$; luminosity distance already contains the cosmological relation between photon energy, arrival rate and geometry; and $H(z)$ links redshift to distance and cosmic time. In a realistic cosmology, luminous-source activity begins at finite cosmic time, so $\mathcal{L}(z)$ is not a non-zero constant extending to an infinite past. The measured EBL is accordingly finite and records the integrated history of stars and galaxies (Dwek and Krennrich, 2013; Hauser and Dwek, 2001). Wesson et al. (1987) showed that finite source lifetime controls the order of magnitude of the EBL in their comparison, with expansion providing an additional factor rather than the sole explanation of darkness. The FLRW resolution is therefore a combined statement about source history, geometry and redshift, not a single-factor suppression argument.

5 Phenomenological Velocity-Differential Redshift

5.1 Definition and Physical Status of the Postulate

The alternative toy model uses the radial velocity field

$$u(r) = -\sqrt{\frac{2GM}{r}},$$

where M is a characteristic mass and r is radial distance from the domain centre. Its gradient magnitude is

$$\left|\frac{du}{dr}\right| = \frac{\sqrt{2GM}}{2} r^{-3/2}.$$

Following the velocity-differential proposal only as a phenomenological rule (Ranzan, 2014), assume

$$\frac{d\ln\lambda}{ds} = \frac{1}{c}\left|\frac{du}{dr}\right|,$$

where s is path length. The expression is dimensionally consistent because du/dr has dimensions of inverse time and $c^{-1}du/dr$ has dimensions of inverse length. Dimensional consistency, however, is not a derivation from a covariant field theory. Nothing in this paper establishes that this wavelength law follows from General Relativity, Maxwell theory, quantum electrodynamics or a specified microscopic medium.

5.2 Directionality, Reciprocity and Energy Accounting

The absolute value in the postulate has a direct physical consequence. Along any path segment for which $|du/dr| > 0$, the right-hand side is non-negative, so $\ln\lambda$ increases with accumulated path length. Reversing a trajectory through the same static domain therefore does not automatically invert the wavelength change; the reversed path accumulates another positive increment. The rule is thus non-reciprocal as written.

A non-reciprocal propagation law is not mathematically inconsistent, but it cannot be interpreted as conservative gravitational frequency transport without additional dynamics. If photon energy decreases while the background field remains unchanged, the model must specify where the transferred energy resides and how energy-momentum conservation is implemented. Alternatively, a signed derivative rather than an absolute value could generate different behaviour, but that would constitute a different model and is not substituted here. The present calculation therefore retains the stated postulate and treats non-reciprocity and energy transfer as explicit physical requirements that any future mechanism must resolve.

The same issue arises for non-radial trajectories. The scalar rule $|du/dr|$ is defined with respect to a radial profile, whereas a general propagation law would need a coordinate-independent prescription involving the photon trajectory and local fields. No such covariant extension is assumed in the present work.

5.3 Single Radial Domain

For a monotonic radial trajectory with $ds = |dr|$ from emission radius r_{em} to observer radius $r_{\mathrm{obs}} > r_{\mathrm{em}}$,

$$\ln(1+z_{\mathrm{VD}}) = \frac{1}{c}\int_{r_{\mathrm{em}}}^{r_{\mathrm{obs}}} \frac{\sqrt{2GM}}{2} r^{-3/2}\,dr = \frac{\sqrt{2GM}}{c}\left(\frac{1}{\sqrt{r_{\mathrm{em}}}} - \frac{1}{\sqrt{r_{\mathrm{obs}}}}\right).$$

Thus

$$1+z_{\mathrm{VD}} = \exp\left[\frac{\sqrt{2GM}}{c}\left(\frac{1}{\sqrt{r_{\mathrm{em}}}} - \frac{1}{\sqrt{r_{\mathrm{obs}}}}\right)\right].$$

Define

$$\beta = \frac{\sqrt{2GM}}{c\sqrt{r_{\mathrm{em}}}} = \sqrt{\frac{r_s}{r_{\mathrm{em}}}}, \qquad r_s = \frac{2GM}{c^2},$$

where r_s is used only as a convenient length scale. With $x = r_{\mathrm{obs}}/r_{\mathrm{em}}$,

$$1+z_{\mathrm{VD}} = \exp\left[\beta\left(1 - x^{-1/2}\right)\right].$$

As $x \to \infty$,

$$1+z_{\mathrm{VD},\infty} = e^{\beta},$$

which is finite for finite M and r_{em}.

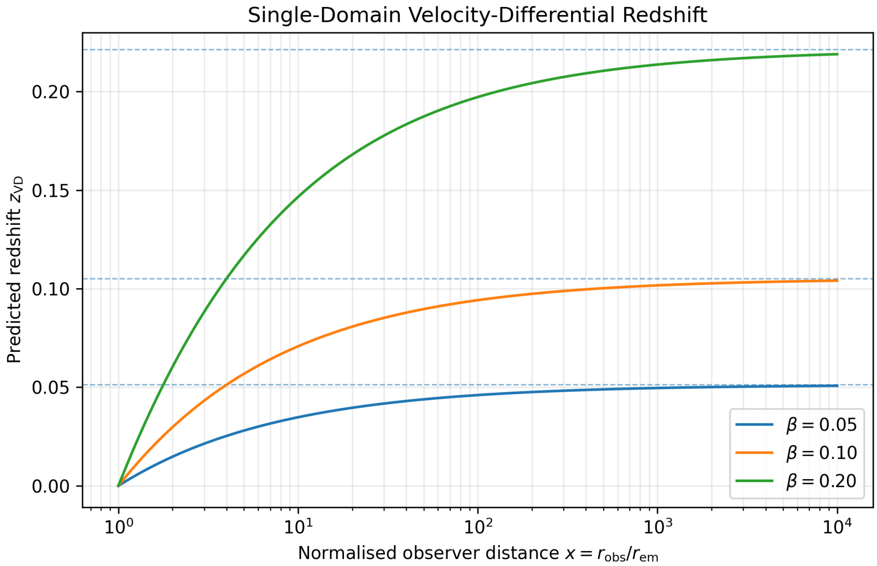

Fig. 1. Dimensionless single-domain velocity-differential redshift as a function of normalised observer distance $x = r_{\text{obs}}/r_{\text{em}}$. Curves use $\beta = 0.05, 0.10$ and 0.20; dashed horizontal lines show the corresponding finite asymptotes $z_\infty = e^\beta - 1$. The figure is illustrative and is not an observational fit

The finite asymptote is decisive for the Olbers test: a single domain cannot generate suppression that increases without bound as propagation distance increases.

6 Generalised Static-Propagation Flux Integral

6.1 Separating Energy Redshift from Arrival-rate Dilation

A propagation-induced redshift need not have the same flux consequences as cosmological redshift. To avoid importing FLRW time-dilation physics into a non-FLRW model, define the observed flux from one static-Euclidean source as

$$F_{\text{obs}}(r) = \frac{L}{4\pi r^2} e^{-\tau(r)} [1 + z(r)]^{-q}.$$

For $q = 1$, the factor represents photon-energy reduction only. For $q = 2$, an additional factor $(1 + z)^{-1}$ represents arrival-rate dilation. Values outside these two cases may be treated mathematically, but require independent physical justification.

For constant source density n,

$$dF = nL\, e^{-\tau(r)} [1 + z(r)]^{-q}\, dr,$$

and therefore

$$F = nL \int_0^\infty e^{-\tau(r)} [1 + z(r)]^{-q}\, dr.$$

Within this toy model, the necessary and sufficient convergence condition is

$$\int_0^\infty e^{-\tau(r)} [1 + z(r)]^{-q} \, dr < \infty.$$

6.2 Single-domain Asymptotics

For the single-domain result, $1 + z(r) \to e^\beta$. If $\tau(r) = 0$,

$$[1 + z(r)]^{-q} \to e^{-q\beta} > 0,$$

so

$$F \sim nLe^{-q\beta} \int^\infty dr \to \infty.$$

The conclusion is independent of whether $q = 1$, $q = 2$ or any other finite positive value. A finite asymptotic redshift leaves a non-zero shell contribution and therefore cannot regularise an infinite static source population.

6.3 Deterministic Cumulative Multi-domain Ansatz

Suppose a photon crosses N domains and the ith domain contributes a multiplicative wavelength factor $1 + \delta_i > 0$. Then

$$1 + z = \prod_{i=1}^{N} (1 + \delta_i), \qquad \ln(1 + z) = \sum_{i=1}^{N} \ln(1 + \delta_i).$$

If one assumes that the cumulative large-scale behaviour is deterministic and exponential,

$$1 + z(r) = e^{\alpha r}, \qquad \alpha > 0,$$

and writes $\tau(r) = \kappa r$, then

$$F = nL \int_0^\infty e^{-(\kappa + q\alpha)r} \, dr = \frac{nL}{\kappa + q\alpha},$$

provided

$$\kappa + q\alpha > 0.$$

The mathematical result is exact under the stated ansatz. The physical inference is limited because the exponential law is not derived from the single-domain field. In particular, domain boundaries, impact parameters, orientations and possible blueshift contributions have not been specified.

6.4 Stochastic Multi-domain Generalisation

The deterministic exponential law can describe a typical path under restrictive statistical assumptions, but the Olbers problem concerns an integrated radiation field and therefore requires attention to ensemble averages. Let

$$Y_i = \ln(1 + \delta_i),$$

and assume, for the present calculation, that the Y_i are independent and identically distributed with a finite transform

$$m_q = E(e^{-qY}) = E[(1 + \delta)^{-q}].$$

For a fixed domain count N, the redshift suppression factor is

$$S_N = e^{-q \sum_{i=1}^{N} Y_i},$$

and independence gives

$$E(S_N) = m_q^N.$$

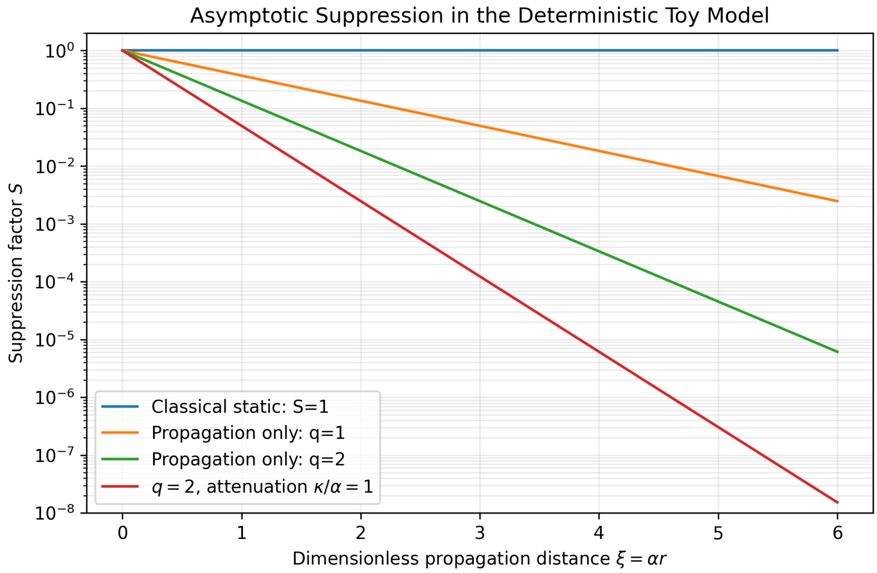

Fig. 2. Dimensionless suppression factor in the deterministic static-Euclidean toy model. The horizontal coordinate is $\xi = \alpha r$. Curves show the classical case $S = 1$, propagation-only cases with $q = 1$ and $q = 2$, and an illustrative combined case with $q = 2$ and $\kappa/\alpha = 1$. The figure depicts analytical behaviour rather than observational fitting

If $N(r) \simeq \eta r$ deterministically at large distance, then

$$E[S(r)] \simeq \exp[\eta r \ln m_q].$$

Including $e^{-\kappa r}$ attenuation, the ensemble-mean shell contribution is asymptotically proportional to

$$\exp[-(\kappa - \eta \ln m_q)r],$$

so the fixed-density stochastic model converges when

$$\kappa - \eta \ln m_q > 0.$$

Without attenuation, this reduces to $m_q < 1$.

If instead $N(r)$ is Poisson with mean ηr, conditioning on N gives

$$E[S(r)] = \exp[\eta r(m_q - 1)],$$

and therefore the ensemble-mean flux converges when

$$\kappa + \eta(1 - m_q) > 0.$$

Again, for $\kappa = 0$, the requirement is $m_q < 1$.

This distinction is important. A positive mean logarithmic increment $E(Y) > 0$ supports typical-path redshift growth through a law-of-large-numbers argument, but it does not by itself ensure $m_q < 1$. Rare negative-Y events, corresponding to blueshift increments, can contribute disproportionately to $E(e^{-qY})$. Consequently, the sign of $E(Y)$ is insufficient to establish convergence of the ensemble-mean background. A physically developed multi-domain theory would have to predict the full distribution of δ, its correlations, and the domain-count process.

Table 1. Analytical convergence summary. The FLRW row uses cosmological distances and evolving comoving emissivity and is not identical to the static-Euclidean toy-model rows

Framework	Large-distance form	Relevant suppression	Convergence statement
Classical static	$z = 0, \tau = 0$	Shell contribution tends to a non-zero constant	Divergent
Single velocity-differential domain	$1 + z \to e^{\beta}$	$[1 + z]^{-q} \to e^{-q\beta} > 0$	Divergent
Deterministic cumulative propagation	$1 + z = e^{\alpha r}$	$e^{-q\alpha r}$	Convergent for $q\alpha > 0$
Deterministic cumulative plus attenuation	$1 + z = e^{\alpha r}, \tau = \kappa r$	$e^{-(\kappa+q\alpha)r}$	Convergent for $\kappa + q\alpha > 0$
Stochastic domains, fixed density	$N \simeq \eta r$	$E[S] \simeq e^{\eta r \ln m_q}$	Convergent for $\kappa - \eta \ln m_q > 0$
Stochastic domains, Poisson count	$N \sim \text{Poisson}(\eta r)$	$E[S] = e^{\eta r (m_q - 1)}$	Convergent for $\kappa + \eta(1 - m_q) > 0$
FLRW	$I_{\text{bol}} \propto \int \mathcal{L}(z)[H(z)(1 + z)^2]^{-1} dz$	Finite source history plus cosmological distance/redshift factors	Finite for realistic emissivity histories

7 Observable Predictions and Discriminants

Mathematical convergence is only a necessary condition for a candidate cosmological explanation. A propagation law becomes physically informative only when it predicts independently measurable quantities. The deterministic exponential ansatz is therefore used below to derive discriminants; these derivations do not constitute evidence that the ansatz is realised in nature.

7.1 Redshift-distance Relation and Luminosity-distance Curvature

If

$$1 + z = e^{\alpha r},$$

then

$$r(z) = \frac{\ln(1 + z)}{\alpha}.$$

With negligible attenuation, the static-Euclidean flux prescription can be written in terms of an effective luminosity distance

$$D_{L,\text{eff}}(z) = r(z)(1 + z)^{q/2} = \frac{\ln(1 + z)}{\alpha}(1 + z)^{q/2}.$$

At low redshift,

$$D_{L,\text{eff}}(z) = \frac{1}{\alpha}\left[z + \frac{q - 1}{2}z^2 + O(z^3)\right].$$

Matching the leading Hubble slope gives $\alpha \simeq H_0/c$. The standard cosmographic luminosity-distance expansion is

$$D_L(z) = \frac{c}{H_0}\left[z + \frac{1 - q_0^{\text{dec}}}{2}z^2 + O(z^3)\right],$$

where q_0^{dec} is the present deceleration parameter. Therefore, in the attenuation-free toy model, matching the first two coefficients requires

$$q = 2 - q_0^{\mathrm{dec}}.$$

As a benchmark rather than a model-independent inference, Pantheon+ gives $\Omega_M = 0.334 \pm 0.018$ for flat ΛCDM from supernovae alone (Brout et al., 2022). In flat ΛCDM, $q_0^{\mathrm{dec}} = 1.5\Omega_M - 1$, giving approximately $q_0^{\mathrm{dec}} = -0.499 \pm 0.027$ and hence a second-order matching value $q \simeq 2.499 \pm 0.027$. Thus the simple $q = 1$ and $q = 2$ versions of the attenuation-free exponential toy model do not reproduce even the benchmark second-order luminosity-distance curvature. This is not a substitute for a full likelihood fit, but it provides a direct analytic falsifiability test.

The same mapping yields

$$\frac{dr}{dz} = \frac{1}{\alpha(1+z)}.$$

If this derivative is interpreted as an effective radial comoving-distance relation $dr/dz = c/H_{\mathrm{eff}}(z)$, then

$$H_{\mathrm{eff}}(z) = c\alpha(1+z),$$

or $H_{\mathrm{eff}}(z) = H_0(1+z)$ after low-redshift normalisation. BAO measurements constrain transverse and radial distances over a wide redshift range and therefore test this functional form independently. The completed eBOSS programme provides BAO measurements from multiple tracers and redshift intervals and shows the constraining power of these independent distance measures (Alam et al., 2021). A viable propagation cosmology would have to reproduce the transverse and radial BAO observables simultaneously, not merely the low-redshift Hubble slope.

7.2 Time Dilation and the Physical Meaning of q

Type Ia supernova light curves exhibit the temporal broadening expected from cosmological expansion. Goldhaber et al. (2001) demonstrated the $(1+z)$ time-axis broadening in earlier supernova data. A substantially larger Dark Energy Survey analysis of 1504 Type Ia supernovae over approximately $0.1 \lesssim z \lesssim 1.2$ found the time-dilation exponent $b = 1.003 \pm 0.005$ (statistical) ± 0.010 (systematic) for $\Delta t_{\mathrm{obs}} = \Delta t_{\mathrm{em}}(1+z)^b$ (White et al., 2024).

This observation has a direct bearing on the toy-model exponent. An energy-only mechanism with unchanged arrival intervals corresponds to $q = 1$ and does not reproduce the observed time dilation as a complete explanation of cosmological redshift. Setting $q = 2$ cannot be justified merely because it improves flux suppression; the same propagation dynamics must derive the observed stretching of arrival times. Thus supernova time dilation is not an optional secondary test but a necessary consistency condition for any non-expansion mechanism that seeks to use $q = 2$.

7.3 Surface Brightness

In FLRW cosmology, bolometric surface brightness obeys the Tolman $(1+z)^{-4}$ dimming relation, with observational applications requiring corrections for luminosity evolution, bandpass and selection. Lubin and Sandage (2001) found results consistent with expansion after accounting for luminosity evolution and reported strong inconsistency with a simple tired-light interpretation.

The present static-Euclidean toy model makes a different prediction unless additional angular physics is supplied. For a resolved source of fixed physical emitting area, Euclidean angular area scales as r^{-2} while the total flux scales as $r^{-2}(1+z)^{-q}$, so surface brightness scales as

$$B_{\mathrm{obs}} \propto (1+z)^{-q}.$$

The elementary $q = 1$ and $q = 2$ cases therefore predict $(1 + z)^{-1}$ and $(1 + z)^{-2}$ dimming, respectively, rather than the FLRW bolometric $(1 + z)^{-4}$ law. A modified propagation theory could alter angular-size relations, but those changes would have to be derived rather than appended after the fact. Surface brightness is therefore an additional independent discriminator.

7.4 CMB Spectral Preservation

The COBE/FIRAS measurement found the CMB spectrum to be extremely close to a blackbody and placed stringent limits on spectral distortions (Fixsen et al., 1996). A cumulative wavelength-shift process acting over cosmological path lengths must specify whether it is achromatic, how it transforms photon phase-space occupation numbers, and how any transferred energy is accounted for. A uniform frequency rescaling alone is not sufficient to guarantee blackbody preservation unless the associated transformation of intensity and phase-space density is dynamically consistent. A mechanism that generated observable distortions beyond the FIRAS limits would be excluded regardless of whether it regularised an Olbers-type integral.

7.5 Diffuse Background Light and Opacity

The optical and infrared EBL constrains the integrated radiative output of cosmic sources, while gamma-ray attenuation probes the intervening background photon density. Dwek and Krennrich (2013) review the relation between EBL intensity and gamma-ray opacity. The Fermi-LAT Collaboration (2018) used attenuation in 739 active galaxies and one gamma-ray burst to reconstruct EBL evolution and the cosmic star-formation history over much of cosmic time. These measurements imply that α and κ cannot be chosen solely to enforce mathematical convergence. Their magnitude and wavelength dependence must remain compatible with independently measured diffuse backgrounds and gamma-ray optical depths.

8 Discussion

The analysis separates four statements that can otherwise be conflated. First, the classical static Olbers construction diverges because each Euclidean shell contributes the same bolometric flux under its idealised assumptions. Second, standard FLRW cosmology does not use that construction: source history, luminosity distance, comoving volume, redshift and cosmic time are linked geometrically, producing a finite observed background for realistic emissivity histories. Third, the velocity-differential postulate examined here yields only a finite redshift in a single $r^{-1/2}$ domain and therefore fails to resolve the static divergence. Fourth, convergence in a cumulative propagation model depends on additional assumptions that are not contained in the single-domain calculation.

The single-domain non-convergence result is the strongest conclusion that follows directly from the stated postulate. It does not depend on observational calibration, on a choice between $q = 1$ and $q = 2$, or on a particular domain-count law. Any finite asymptotic redshift leaves a non-zero shell contribution at large distance and hence preserves the divergence.

The stochastic-domain extension sharpens the status of the cumulative construction. Writing $\alpha = \eta E(Y)$ may describe the logarithmic redshift of a typical long path when the assumptions of a law of large numbers apply. However, the radiation background depends on averages of the flux-suppression factor, which involve $E[e^{-qY}]$ rather than $E(Y)$ alone. This distinction matters because the exponential function weights rare negative-Y events strongly. A positive average redshift increment is therefore not sufficient to prove convergence of the mean background. The full increment distribution and its correlations are part of the physical model, not technical details that can be omitted.

The reciprocity issue provides a second important constraint. Because the postulate contains $|du/dr|$, the wavelength shift is positive for either traversal direction through a static gradient. Such non-reciprocity may be permissible in a dissipative theory, but then the theory must identify the reservoir receiving energy and must formulate the process in a manner consistent with energy-momentum accounting. A conservative gravitational interpretation cannot simply be assumed.

The observational comparisons further narrow the possible interpretation. In the simplest attenuation-free exponential law, the luminosity-distance series implies a specific relation between the flux exponent q and the observed second-order distance curvature. Using Pantheon+ flat-ΛCDM parameters only as a benchmark, the required value is approximately $q \simeq 2.5$, rather than either elementary case $q = 1$ or $q = 2$. The same model predicts $H_{\text{eff}}(z) \propto (1+z)$ for radial distance and surface-brightness dimming $(1 + z)^{-q}$ under Euclidean angular geometry. It must also reproduce the well-measured supernova time-dilation relation. These requirements are independent of the Olbers convergence calculation and can falsify the propagation model even when the integral itself converges.

The role of the Ranzan (2014) paper is therefore limited to provenance of the tested phenomenological postulate. The present analysis neither adopts the source paper's luminiferous-substrate interpretation nor treats it as established physics. A viable alternative cosmology would require a covariant or otherwise precisely specified dynamics, a complete treatment of photon propagation and angular observables, conservation laws, and quantitative fits to independent datasets. The toy model is useful primarily because it makes these requirements explicit.

9 Limitations

Several limitations define the scope of the results. First, the wavelength-evolution law is postulated rather than derived from an action, metric theory or microscopic interaction. Second, the use of $|du/dr|$ makes the law non-reciprocal as written, and the physical reservoir associated with photon-energy change is unspecified. Third, the single-domain calculation is radial; no coordinate-independent non-radial extension is supplied. Fourth, the deterministic multi-domain relation $1 + z = e^{\alpha r}$ is an ansatz rather than a consequence of the single-domain field. Fifth, the stochastic treatment assumes independent, identically distributed increments and uses idealised fixed-density or Poisson domain counts; real structures could introduce correlations, environment dependence and broad domain-property distributions. Sixth, the exponent q remains phenomenological because the propagation dynamics do not derive arrival-time dilation. Seventh, κ is an effective attenuation term and is not a universal grey opacity or a complete bolometric energy sink. Eighth, the observational comparison is analytic rather than a full likelihood analysis of supernova, BAO, CMB, surface-brightness and EBL datasets. The cumulative propagation construction should therefore be regarded as a falsifiable mathematical toy model, not an empirically established alternative cosmology.

10 Conclusion

Olbers' paradox is fundamentally a convergence problem, but the correct convergence calculation depends on the cosmological framework. In a static Euclidean universe with constant source density and luminosity, every radial shell contributes the same flux and the integrated background diverges. In FLRW cosmology, the bolometric background instead depends on comoving luminosity density, luminosity distance, comoving volume, expansion history and finite source history, so the assumptions of the classical paradox do not apply.

For the phenomenological field $u(r) = -\sqrt{2GM/r}$ and the postulated relation $d \ln \lambda / ds = c^{-1} |du/dr|$, the single-domain redshift approaches a finite value $1 + z_\infty = e^\beta$. The corresponding static-Euclidean Olbers integral therefore remains divergent for every finite positive q in the absence of another distance-dependent suppression. This negative result is robust within the stated assumptions.

A deterministic cumulative law $1+z = e^{\alpha r}$ is mathematically sufficient to yield finite flux when $\kappa + q\alpha > 0$, but it is not derived from the single-domain dynamics. The stochastic extension strengthens this conclusion: without attenuation, ensemble-mean convergence requires $m_q = E[(1+\delta)^{-q}] < 1$, not merely a positive mean logarithmic redshift increment. Thus the distribution of domain effects, including possible blueshift tails, is essential.

The same toy model is strongly constrained by observations outside the Olbers problem. Its luminosity-distance curvature, radial distance derivative, time-dilation behaviour, surface-brightness law, CMB spectral transformation and EBL opacity must all be reproduced by one consistent mechanism. Present observations already make these tests stringent. Accordingly, the propagation construction is best understood as a transparent hypothesis-testing device that identifies necessary mathematical and observational conditions, rather than as a replacement for FLRW/ΛCDM.

Declaration of AI Use

This manuscript was prepared through the intellectual contributions of the author(s) to the study design, data analysis, interpretation of results, and scholarly content. Grammarly and ChatGPT were used solely to assist with grammatical refinement and reference formatting during manuscript revision. The author(s) accept full responsibility for the accuracy, integrity, and final content of the manuscript.

Competing Interests

Author has declared that no competing interests exist.

References

Alam, S., Aubert, M., Avila, S., Balland, C., Bautista, J. E., Bershady, M. A., Bizyaev, D., Blanton, M. R., Bolton, A. S., Bovy, J., Brinkmann, J., Brownstein, J. R., Burtin, E., Chabanier, S., Chapman, M. J., Choi, P. D., Chuang, C.-H., Comparat, J., Cousinou, M.-C., and Zheng, Z. (2021). Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: Cosmological implications from two decades of spectroscopic surveys at the Apache Point Observatory. *Physical Review D*, 103(8):083533. https://doi.org/10.1103/PhysRevD.103.083533.

Brout, D., Scolnic, D., Popovic, B., Riess, A. G., Carr, A., Zuntz, J., Kessler, R., Davis, T. M., Hinton, S., Jones, D., D'Arcy Kenworthy, W., Peterson, E. R., Said, K., Taylor, G., Ali, N., Armstrong, P., Charvu, P., Dwomoh, A., Meldorf, C., and Wiseman, P. (2022). The Pantheon+ analysis: Cosmological constraints. *The Astrophysical Journal*, 938(2):110. https://doi.org/10.3847/1538-4357/ac8e04.

Dwek, E. and Krennrich, F. (2013). The extragalactic background light and the gamma-ray opacity of the universe. *Astroparticle Physics*, 43:112–133. https://doi.org/10.1016/j.astropartphys.2012.09.003.

Etherington, I. M. H. (2007). Republication of: LX. on the definition of distance in general relativity. *General Relativity and Gravitation*, 39(7):1055–1067. https://doi.org/10.1007/s10714-007-0447-x.

Fermi-LAT Collaboration (2018). A gamma-ray determination of the Universe's star formation history. *Science*, 362(6418):1031–1034. https://doi.org/10.1126/science.aat8123.

Fixsen, D. J., Cheng, E. S., Gales, J. M., Mather, J. C., Shafer, R. A., and Wright, E. L. (1996). The cosmic microwave background spectrum from the full COBE FIRAS data set. *The Astrophysical Journal*, 473(2):576–587. https://doi.org/10.1086/178173.

Frieman, J. A., Turner, M. S., and Huterer, D. (2008). Dark energy and the accelerating universe. *Annual Review of Astronomy and Astrophysics*, 46:385–432. https://doi.org/10.1146/annurev.astro.46.060407.145243.

Goldhaber, G., Groom, D. E., Kim, A., Aldering, G., Astier, P., Conley, A., Deustua, S. E., Ellis, R., Fabbro, S., Fruchter, A. S., Goobar, A., Hook, I., Irwin, M., Kim, M., Knop, R. A., Lidman, C., McMahon, R., Nugent, P. E., Pain, R., and York, T. (2001). Timescale stretch parameterization of Type Ia supernova B-band light curves. *The Astrophysical Journal*, 558(1):359–368. https://doi.org/10.1086/322460.

Harrison, E. R. (1964). Olbers' paradox. *Nature*, 204:271–272. https://doi.org/10.1038/204271b0.

Hauser, M. G. and Dwek, E. (2001). The cosmic infrared background: Measurements and implications. *Annual Review of Astronomy and Astrophysics*, 39:249–307. https://doi.org/10.1146/annurev.astro.39.1.249.

Lubin, L. M. and Sandage, A. (2001). The Tolman surface brightness test for the reality of the expansion. IV. A measurement of the Tolman signal and the luminosity evolution of early-type galaxies. *The Astronomical Journal*, 122(3):1084–1103. https://doi.org/10.1086/322134.

Planck Collaboration (2020). Planck 2018 results. VI. Cosmological parameters. *Astronomy & Astrophysics*, 641:A6. https://doi.org/10.1051/0004-6361/201833910.

Ranzan, C. (2014). Cosmic redshift in the nonexpanding cellular universe: Velocity-differential theory of cosmic redshift. *American Journal of Astronomy and Astrophysics*, 2(5):47–60. https://doi.org/10.11648/j.ajaa.20140205.11.

Wesson, P. S., Valle, K., and Stabell, R. (1987). The extragalactic background light and a definitive resolution of Olbers' paradox. *The Astrophysical Journal*, 317:601–606. https://doi.org/10.1086/165306.

White, R. M. T., Davis, T. M., Lewis, G. F., Brout, D., Galbany, L., Glazebrook, K., Hinton, S. R., Lee, J., Lidman, C., Möller, A., Sako, M., Scolnic, D., Smith, M., Sullivan, M., Sánchez, B. O., Shah, P., Vincenzi, M., Wiseman, P., and Abbott, T. M. C. (2024). The Dark Energy Survey supernova program: Slow supernovae show cosmological time dilation out to $z \sim 1$. *Monthly Notices of the Royal Astronomical Society*, 533(3):3365–3378. https://doi.org/10.1093/mnras/stae2008.

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